Designation: E29 – 13

An American National Standard

Standard Practice for Using Significant Digits in Test Data to Determine Conformance with Specifications[1]

This standard is issued under the fixed designation E29; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ε) indicates an editorial change since the last revision or reapproval.

This standard has been approved for use by agencies of the Department of Defense.

1. Scope*

1.1 This practice is intended to assist the various technical committees in the use of uniform methods of indicating the number of digits which are to be considered significant in specification limits, for example, specified maximum values and specified minimum values. Its aim is to outline methods which should aid in clarifying the intended meaning of specification limits with which observed values or calculated test results are compared in determining conformance with specifications.

1.2 This practice is intended to be used in determining conformance with specifications when the applicable ASTM specifications or standards make direct reference to this practice.

1.3 Reference to this practice is valid only when a choice of method has been indicated, that is, either *absolute method or rounding method*.

1.4 The system of units for this practice is not specified. Dimensional quantities in the practice are presented only as illustrations of calculation methods. The examples are not binding on products or test methods treated.

1.5 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to use.*

2. Referenced Documents

2.1 *ASTM Standards:*[2]

E177 Practice for Use of the Terms Precision and Bias in ASTM Test Methods

E456 Terminology Relating to Quality and Statistics

E2282 Guide for Defining the Test Result of a Test Method

IEEE/ASTM SI 10 Standard for Use of the International System of Units (SI): The Modern Metric System

3. Terminology

3.1 *Definitions*—Terminology E456 provides a more extensive list of terms in E11 standards.

3.1.1 *observed value, n*—the value obtained by making an observation. **E2282**

3.1.2 *repeatability conditions, n*—conditions where independent test results are obtained with the same method on identical test items in the same laboratory by the same operator using the same equipment within short intervals of time. **E177**

3.1.3 *repeatability standard deviation (s_r), n*—the standard deviation of test results obtained under repeatability conditions. **E177**

3.1.4 *significant digit*—any of the figures 0 through 9 that is used with its place value to denote a numerical quantity to some desired approximation, excepting all leading zeros and some trailing zeros in numbers not represented with a decimal point.

3.1.4.1 *Discussion*—This definition of significant digits relates to how the number is represented as a decimal. It should not be inferred that a measurement value is precise to the number of significant digits used to represent it.

3.1.4.2 *Discussion*—The digit zero may either indicate a specific value or indicate place only. Zeros leading the first nonzero digit of a number indicate order of magnitude only and are not significant digits. For example, the number 0.0034 has two significant digits. Zeros trailing the last nonzero digit for numbers represented with a decimal point are significant digits. For example, the numbers 1270. and 32.00 each have four significant digits. The significance of trailing zeros for numbers represented without use of a decimal point can only be identified from knowledge of the source of the value. For example, a modulus strength, stated as 140 000 Pa, may have as few as two or as many as six significant digits.

[1] This practice is under the jurisdiction of ASTM Committee E11 on Quality and Statistics and is the direct responsibility of Subcommittee E11.30 on Statistical Quality Control.

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[2] For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

***A Summary of Changes section appears at the end of this standard**

3.1.4.3 *Discussion*—To eliminate ambiguity, the exponential notation may be used. Thus, 1.40×10^5 indicates that the modulus is reported to the nearest 0.01×10^5 or 1000 Pa.

3.1.4.4 *Discussion*—Use of appropriate SI prefixes is recommended for metric units to reduce the need for trailing zeros of uncertain significance. Thus, 140 kPa (without the decimal point) indicates that the modulus is reported either to the nearest 10 or 1 kPa, which is ambiguous with respect to the number of significant digits. However, 0.140 MPa clearly indicates that the modulus is reported to the nearest 1 kPa, and 0.14 MPa clearly indicates that the modulus is reported to the nearest 10 kPa.

3.1.5 *test result, n*—the value of a characteristic obtained by carrying out a specified test method. **E2282**

4. Significance and Use

4.1 This practice describes two commonly accepted methods of rounding data, identified as the Absolute Method and the Rounding Method. In the applications of this practice to a specific material or materials it is essential to specify which method is intended to apply. In the absence of such specification, reference to this practice, which expresses no preference as to which method should apply, would be meaningless. The choice of method depends upon the current practice of the particular branch of industry or technology concerned, and should therefore be specified in the prime publication.

4.1.1 The unqualified statement of a numerical limit, such as "2.50 in. max," cannot, in view of different established practices and customs, be regarded as carrying a definite operational meaning concerning the number of digits to be retained in an observed or a calculated value for purposes of determining conformance with specifications.

4.1.2 *Absolute Method*—In some fields, specification limits of 2.5 in. max, 2.50 in. max, and 2.500 in. max are all taken to imply the same absolute limit of exactly two and a half inches and for purposes of determining conformance with specifications, an observed value or a calculated value is to be compared directly with the specified limit. Thus, any deviation, however small, outside the specification limit signifies nonconformance with the specifications. This will be referred to as the *absolute method*, which is discussed in Section 5.

4.1.3 *Rounding Method*—In other fields, specification limits of 2.5 in. max, 2.50 in. max, 2.500 in. max are taken to imply that, for the purposes of determining conformance with specifications, an observed value or a calculated value should be rounded to the nearest 0.1 in., 0.01 in., 0.001 in., respectively, and then compared with the specification limit. This will be referred to as the *rounding method*,which is discussed in Section 6.

4.2 Section 7 of this practice gives guidelines for use in recording, calculating, and reporting the final result for test data.

5. Absolute Method

5.1 *Where Applicable*—The absolute method applies where it is the intent that all digits in an observed value or a calculated value are to be considered significant for purposes of determining conformance with specifications. Under these conditions, the specified limits are referred to as absolute limits.

5.2 *How Applied*—With the absolute method, an observed value or a calculated value is not to be rounded, but is to be compared directly with the specified limiting value. Conformance or nonconformance with the specification is based on this comparison.

5.3 *How Expressed*—This intent may be expressed in the standard in one of the following forms:

5.3.1 If the absolute method is to apply to all specified limits in the standard, this may be indicated by including the following sentence in the standard:

> For purposes of determining conformance with these specification, all specified limits in this standard are absolute limits, as defined in ASTM Practice E29, for Using Significant Digits in Test Data to Determine Conformance with Specifications.

5.3.2 If the absolute method is to apply to all specified limits of some general type in the standard (such as dimensional tolerance limits), this may be indicated by including the following sentence in the standard:

> For purposes of determining conformance with these specifications, all specified (dimensional tolerance) limits are absolute limits, as defined in ASTM Practice E29, for Using Significant Digits in Test Data to Determine Conformance with Specifications.

5.3.3 If the absolute method is to apply to all specified limits given in a table, this may be indicated by including a footnote with the table as follows:

Capacity mL	Volumetric Tolerance[A] ± mL
10	0.02
25	0.03
50	0.05
100	0.10

[A] Tolerance limits specified are absolute limits as defined in Practice E29, for Using Significant Digits in Test Data to Determine Conformance with Specifications.

6. Rounding Method

6.1 *Where Applicable*—The rounding method applies where it is the intent that a limited number of digits in an observed value or a calculated value are to be considered significant for purposes of determining conformance with specifications.

6.2 *How Applied*—With the rounding method, an observed value or a calculated value should be rounded by the procedure prescribed in 4.1.3 to the nearest unit in the designated place of figures stated in the standard, as, for example, "to the nearest kPa," "to the nearest 10 ohms," "to the nearest 0.1 percent," etc. The rounded value should then be compared with the specified limit, and conformance or nonconformance with the specification based on this comparison.

6.3 *How Expressed*—This intent may be expressed in the standard in one of the following forms:

6.3.1 If the rounding method is to apply to all specified limits in the standard, and if all digits expressed in the specification limit are to be considered significant, this may be indicated by including the following statement in the standard:

The following applies to all specified limits in this standard: For purposes of determining conformance with these specifications, an observed value or a calculated value shall be rounded "to the nearest unit" in the last right-hand digit used in expressing the specification limit, in accordance with the rounding method of ASTM Practice E29, for Using Significant Digits in Test Data to Determine Conformance with Specifications.

6.3.2 If the rounding method is to apply only to the specified limits for certain selected requirements, this may be indicated by including the following statement in the standard:

The following applies to specified limits for requirements on (tensile strength), (elongation), and (...) given in ..., (applicable section number and title) and (...) of this standard: For purposes of determining conformance with these specifications, an observed value or a calculated value shall be rounded to the nearest 1kPa for (tensile strength), to the nearest (1 percent) for (elongation), and to the nearest (...) for (...) in accordance with the rounding method of ASTM Practice E29 Using Significant Digits in Test Data to Determine Conformance with Specifications.

6.3.3 If the rounding method is to apply to all specified limits in a table, this may be indicated by a note in the manner shown in the following examples:

6.3.3.1 *Example 1*—Same significant digits for all items:

	Chemical Composition, % mass
Copper	4.5 ± 0.5
Iron	1.0 max
Silicon	2.5 ± 0.5
Other constituents (magnesium + zinc + manganese)	0.5 max
Aluminum	remainder

For purposes of determining conformance with these specifications, an observed value or a calculated value shall be rounded to the nearest 0.1 percent, in accordance with the rounding method of ASTM Practice E29 Using Significant Digits in Test Data to Determine Conformance with Specifications.

6.3.3.2 *Example 2*—Significant digits not the same for all items; similar requirements:

	Chemical Composition, % mass	
	min	max
Nickel	57	...
Chromium	14	18
Manganese	...	3
Silicon	...	0.40
Carbon	...	0.25
Sulfur	...	0.03
Iron	remainder	

For purposes of determining conformance with these specifications, an observed value or a calculated value shall be rounded "to the nearest unit" in the last right-hand significant digit used in expressing the limiting value, in accordance with the rounding method of ASTM Practice E29 Using Significant Digits in Test Data to Determine Conformance with Specifications.

6.3.3.3 *Example 3*—Significant digits not the same for all items; dissimilar requirements:

	Tensile Requirements
Tensile strength, psi	60 000 to 72 000
Yield point, min, psi	33 000
Elongation in 2 in., min %	22

For purposes of determination of conformance with these specifications, an observed value or a calculated value shall be rounded to the nearest 1000 psi for tensile strength and yield point and to the nearest 1 percent for elongation, in accordance with the rounding method of ASTM Practice E29 Using Significant Digits in Test Data to Determine Conformance with Specifications.

6.4 *Rounding Procedure*—The actual rounding procedure[3] shall be as follows:

6.4.1 When the digit next beyond the last place to be retained is less than 5, retain unchanged the digit in the last place retained.

6.4.2 When the digit next beyond the last place to be retained is greater than 5, increase by 1 the digit in the last place retained.

6.4.3 When the digit next beyond the last place to be retained is 5, and there are no digits beyond this 5, or only zeros, increase by 1 the digit in the last place retained if it is odd, leave the digit unchanged if it is even. Increase by 1 the digit in the last place retained, if there are non-zero digits beyond this 5.

NOTE 1—This method for rounding 5's is not universally used by software packages.

6.4.4 This rounding procedure may be restated simply as follows: When rounding a number to one having a specified number of significant digits, choose that which is nearest. If two choices are possible, as when the digits dropped are exactly a 5 or a 5 followed only by zeros, choose that ending in an even digit. Table 1 gives examples of applying this rounding procedure.

6.5 The rounded value should be obtained in one step by direct rounding of the most precise value available and not in two or more successive roundings. For example: 89 490 rounded to the nearest 1 000 is at once 89 000; it would be incorrect to round first to the nearest 100, giving 89 500 and then to the nearest 1 000, giving 90 000.

6.6 *Special Case, Rounding to the Nearest 50, 5, 0.5, 0.05, etc.*—If in special cases it is desired to specify rounding to the nearest 50, 5, 0.5, 0.05, etc., this may be done by so indicating in the standard. In order to round to the nearest 50, 5, 0.5, 0.05, etc., double the observed or calculated value, round to the nearest 100, 10, 1.0, 0.10, etc., in accordance with the procedure in 6.4, and divide by 2. For example, in rounding 6 025 to the nearest 50, 6 025 is doubled giving 12 050 which becomes 12 000 when rounded to the nearest 100 (6.4.3). When 12 000 is divided by 2, the resulting number, 6 000, is the rounded value of 6 025. In rounding 6 075 to the nearest 50,

TABLE 1 Examples[A] of Rounding

Specified Limit	Observed Value or Calculated Value	To Be Rounded to Nearest	Rounded Value to be Used for Purposes of Determining Conformance	Conforms with Specified Limit
Yield point, 36 000 psi, min	35 940	100 psi	35 900	no
	{35 950	100 psi	36 000	yes
	35 960	100 psi	36 000	yes
Nickel, 57 %, mass, min	56.4	1 %	56	no
	{56.5	1 %	56	no
	56.6	1 %	57	yes
Water extract conductivity, 40 ms/m, max	40.4	1 ms/m	40	yes
	{40.5	1 ms/m	40	yes
	40.6	1 ms/m	41	no
Sodium bicarbonate 0.5 %, max, dry mass basis	0.54	0.1 %	0.5	yes
	{0.55	0.1 %	0.6	no
	0.56	0.1 %	0.6	no

[A] These examples are meant to illustrate rounding rules and do not necessarily reflect the usual number of digits associated with these test methods.

[3] The rounding procedure given in this practice is the same as the one given in the *ASTM Manual 7 on Presentation of Data and Control Chart Analysis.*

6 075 is doubled giving 12 150 which becomes 12 200 when rounded to the nearest 100 (6.4.3). When 12 200 is divided by 2, the resulting number, 6 100, is the rounded value of 6 075.

6.7 *Special Case, Rounding to the Nearest Interval Not Covered in 6.4 or 6.6*—In some test methods, there may be a requirement to round a value to an interval that does not align with the specific requirements in 6.4 or 6.6, such as to the nearest 0.02, 0.25, 0.3 etc. In such cases, the following procedure can be used for rounding to any interval:

NOTE 2—Using a calculation subroutine that has been programmed to perform the rounding procedure described in 6.7.1, 6.7.2, and 6.7.3 can be advantageous in evaluating laboratory data.

6.7.1 Divide the result by the desired rounding increment or interval.

6.7.2 Round the result obtained in 6.7.1 to the nearest whole number following the conventions in 6.4, 6.4.1, 6.4.2, and 6.4.3 as appropriate.

6.7.3 Multiply the result obtained in 6.7.2 by the desired rounding increment or interval.

6.7.4 For example, in rounding 0.07 to the nearest 0.02, dividing 0.07 by 0.02 gives a value of 3.5. Rounding this value to the nearest whole number gives a value of 4, based on the information in 6.4.3. Multiplying 4 by 0.02 yields 0.08. In rounding 0.09 to the nearest 0.02, dividing 0.09 by 0.02 gives a value of 4.5. Rounding this value to the nearest whole number gives a value of 4, based on the information in 6.4.3. Multiplying 4 by 0.02 yields 0.08.

7. Guidelines for Retaining Significant Figures in Calculation and Reporting of Test Results

7.1 *General Discussion*—Rounding test results avoids a misleading impression of precision while preventing loss of information due to coarse resolution. Any approach to retention of significant digits of necessity involves some loss of information; therefore, the level of rounding should be carefully selected considering both planned and potential uses for the data. The number of significant digits must, first, be adequate for comparison against specification limits (see 6.2). The following guidelines are intended to preserve the data for statistical summaries. For certain purposes, such as where calculations involve differences of measurements close in magnitude, and for some statistical calculations, such as paired t-tests, autocorrelations, and nonparametric tests, reporting data to a greater number of significant digits may be advisable.

7.2 *Recording Observed Values*—When recording direct measurements, as in reading marks on a buret, ruler, or dial, all digits known exactly, plus one digit which may be uncertain due to estimation, should be recorded. For example, if a buret is graduated in units of 0.1 mL, then an observed value would be recorded as 9.76 mL where it is observed between 9.7 and 9.8 marks on the buret, and estimated about six tenths of the way between those marks. When the measuring device has a vernier scale, the last digit recorded is the one from the vernier.

7.2.1 The number of significant digits given by a digital display or printout from an instrument should be greater than or equal to those given by the rule for reporting test results in 7.4 below.

7.3 *Calculation of Test Result from Observed Values*—When calculating a test result from observed values, avoid rounding of intermediate quantities. As far as is practicable with the calculating device or form used, carry out calculations with the observed values exactly and round only the final result.

7.4 *Reporting Test Results*—A suggested rule relates the significant digits of the test result to the precision of the measurement expressed as the standard deviation σ. The applicable standard deviation is the repeatability standard deviation. The rounding interval for test results should not be greater than 0.5 σ nor less than 0.05 σ, but not greater than the unit in the specification (see 6.2). When only an estimate, *s*, is available for σ, *s* may be used in place of σ in the preceding sentence. An alternative statement of the suggested rule is: Write down the standard deviation. Round test results to the place of the first significant digit in the standard deviation if the digit is two or higher, to the next place if it is a one.

Example:
A test result is calculated as 1.45729. The standard deviation of the test method is estimated to be, 0.0052. Round to 1.457 or the nearest 0.001 since this rounding unit, 0.001, is between 0.05 σ = 0.00026 and 0.5σ = 0.0026.

NOTE 3—A rationale for this rule is derived from Sheppard's adjustment for grouping, which represents the standard deviation of a rounded test result by $\sqrt{\sigma^2 + w^2/12}$ where σ is the standard deviation of the unrounded test result and *w* is the rounding interval. The quantity $w/\sqrt{12}$ is the standard deviation of an error uniformly distributed over the range *w*. Rounding so that *w* is below 0.5 σ ensures that the standard deviation is increased by at most 1 %.

7.4.1 When no estimate of the standard deviation σ is known, then rules for retention of significant digits of computed quantities may be used to derive a number of significant digits to be reported, based on significant digits of test data.

7.4.1.1 The rule when adding or subtracting test data is that the result contains no significant digits beyond the place of the last significant digit of any datum.

Examples:
(*1*) 11.24 + 9.3 + 6.32 = 26.9, since the last significant digit of 9.3 is the first following the decimal place, 26.9 is obtained by rounding the exact sum, 26.86, to this place of digits.
(*2*) 926 – 923.4 = 3
(*3*) 140 000 + 91 460 = 231 000 when the first value was recorded to the nearest thousand.

7.4.1.2 The rule when multiplying or dividing is that the result contains no more significant digits than the value with the smaller number of significant digits.

Examples:
(*1*) 11.38 × 4.3 = 49, since the factor 4.3 has two significant digits.
(*2*) (926 – 923.4)/4.3 = 0.6 Only one figure is significant since the numerator difference has only one significant digit.

7.4.1.3 The rules for logarithms and exponentials are: Digits of $ln(x)$ or $\log_{10}(x)$ are significant through the n-th place after the decimal when x has *n* significant digits. The number of significant digits of e^x or 10^x is equal to the place of the last significant digit in x after the decimal.

Examples:
ln(3.46) = 1.241 to three places after the decimal, since 3.46 has three significant digits. $10^{3.46}$ = 2900 has two significant digits, since 3.46 is given to two places after the decimal.

7.4.1.4 The rule for numbers representing exact counts or mathematical constants is that they are to be treated as having an infinite number of significant digits.

Examples:
(*1*) 1 – 0.23/2 = 0.88 where the numbers 1 and 2 are exact and 0.23 is an approximate quantity.
(*2*) A count of 50 pieces times a measured thickness 0.124 mm is 50 × 0.124 = 6.20 mm, having three significant figures.
(*3*) A measurement of 1.634 in. to the nearest thousandth, is converted to mm. The result, 1.634 × 25.4 = 41.50 mm, has four significant digits. The conversion constant, 25.4, is exact.

NOTE 4—More extensive discussion of dimensional conversion can be found in IEEE/ASTM SI 10.

7.5 *Specification Limits*—When the rounding method is to apply to given specified limits, it is desirable that the significant digits of the specified limits should conform to the precision of the test following the rule of 7.4. That is, the rounding unit for the specification limits should be between 0.05 and 0.5 times the standard deviation of the test.

7.6 *Averages and Standard Deviations*—When reporting the average and standard deviation of replicated measurements or repeated samplings of a material, a suggested rule for most cases is to round the standard deviation to two significant digits and round the average to the same last place of significant digits. When the number of observations is large (more than 15 when the lead digit of the standard deviation is 1, more than 50 with lead digit 2, more than 100 in other cases), an additional digit may be advisable.

7.6.1 Alternative approaches for averages include reporting $\bar{x}$ to within 0.05 to 0.5 times the standard deviation of the average $\sigma/\sqrt{n}$, or applying rules for retaining significant digits to the calculation of $\bar{x}$. ASTM Manual 7 provides methods for reporting $\bar{x}$ and s for these applications.[3]

NOTE 5—A rationale for the suggested rule comes from the uncertainty of a calculated standard deviation s. The standard deviation of s based on sampling from a normal distribution with n observations is approximately $\sigma/\sqrt{2n}$. Reporting s to within 0.05 to 0.5 of this value, following the rule of 7.4, leads to two significant digits for most values of σ when the number of observations n is 100 or fewer.

Example: Analyses on six specimens give values of 3.56, 3.88, 3.95, 4.07, 4.21, and 4.47 for a constituent. The average and standard deviation, unrounded, are $\bar{x}$ = 4.0233... and s = 0.3089... . The suggested rule would report $\bar{x}$ and s as 4.02 and 0.31.

8. Keywords

8.1 absolute method; conformance; rounding; significant digits; specifications; test data

SUMMARY OF CHANGES

Committee E11 has identified the location of selected changes to this standard since the last issue (E29 – 08) that may impact the use of this standard. (Approved Aug. 1, 2013.)

(1) Revised 7.4, 7.4.1.1, 7.4.1.2, and Note 3.

(2) Added 1.4, 1.5, and Section 8.